Heuristic Model for Vehicle Routing Problem with Time Constrained Based on Genetic Algorithm

Sang-Cheol Lee\textsuperscript{1} and Jeong-Cheol Yu\textsuperscript{2}

Abstract A vehicle routing problem with time constraint is one of the important problems in distribution and transportation. The service of a customer must start and finish within a given time interval. Our method is based on an improved operators of genetic algorithm and the objective is to minimize the cost of servicing the set of customers without being tardy or exceeding the capacity or travel time of the vehicles. This research shows that a proposed method based on the improved genetic search can obtain good solutions to vehicle routing problems with time constrained compared with a high degree of efficiency other heuristics. For the computational purpose, we developed a GUI-type computer program according to the proposed method and the computational results show that the proposed method is very effective on a set of standard test problems, and can be potentially useful in solving the vehicle routing problems.

Key Words : Vehicle Routing Problems, Genetic Algorithm, GA-TSP

1. Introduction

This paper is concerned with a vehicle routing problem with time constrained based on an improved genetic algorithm(GA). In this research, we proposed an improved genetic algorithm for the vehicle routing problem where the improvement is obtained by the method of initial population and crossover operators. The genetic algorithm is based on the mechanics of natural selection and natural genetics. They combine survival of the string structures with structured and randomized information exchange to form a search algorithm. In every generation, a new set of artificial chromosomes is created using pieces of the chromosomes of previous generation. Genetic algorithm generally provides a good solution at slower speed. Generally a simple GA is composed of three operators which are applied sequentially to the current population of chromosomes and interactively generation after generation. These three operators are reproduction, crossover and mutation.

In this research, we proposed an extended GA for TSP-type problem by improving the search progress from population of solution and operators such as crossover...
and mutation. The crossover mechanism of this improved method is based on edge-2 recombination and reciprocal exchange mutation and inversion. This research consists of two steps as shown in Figure 1.

**Figure 1.** Two Steps of Resume

2. Modifications in GA Operators

2.1 Modifications in Operators

This paper describes some modified features of crossover against premature convergence and better solution applied to genetic solutions of the TSP-type problems. There are several basic approaches for genetic algorithm and its operators. These include: Order Crossover (OX), Partially Mapped Crossover (PMX), Cycle Crossover (CX) and Edge Recombination Crossover (ERC) [4, 5].

In this paper, we proposed a modification from two edge crossover recombination crossover. As the result of their applications, the frequency of obtaining an optimal solution was improved with keeping good convergence. This form of algorithm includes reproduction and selection. Each offspring's fitness value is used as a criterion to perform the selection of the potential new parents. From the initial population, first we derived the fitness values of each individual using the function and we sort these values by the ascending order. We applied the edge-2 recombination operator using edge table as a adjacency table listing the edges into and out of a city as observed in the two parent tours. The edge list for some city k is composed of all of the cities in the two parents that are adjacent to k. If some city is adjacent to k in both parents, this entry is marked “-“and it is selected first to construct offspring. The edge recombination operator uses edge table to construct and offspring inherits as much information as possible from the parent structures. The procedure for this proposed approach is listed below.

**Step 1:** Assume that there are two strings of chromosomes string P1 and P2 as parent.

P1 = [ g d h b f a e c ]
P2 = [ c e a g b h f d ]

**Step 2:** Constructs edge table.(Table 1 Edge Table)

**Step 3:** Select cities with a few connections from the edge table.

(from the edge table, e or k characters)

**Step 4:** By sorting the selected characters by alphabetic order, construct new offspring 01, 02.

01 = [ e a f b h d c g ]
02 = [ a e c d f b h g ]

**Step 5:** The flagged cities have first priority, and cities whose own edge list has the fewest entries have second priority.

(Selected cities is remove from edge list)

**Step 6:** This processing continues until the tour is constructed.

(repeat Step 4)

Table 1. shows the Edge Table of improved edge recombination operator.

<table>
<thead>
<tr>
<th>City</th>
<th>Edge List</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>f, e, g</td>
</tr>
<tr>
<td>b</td>
<td>f, g, h</td>
</tr>
<tr>
<td>c</td>
<td>d, -e, g</td>
</tr>
<tr>
<td>d</td>
<td>c, f, g, h</td>
</tr>
<tr>
<td>e</td>
<td>-a, -c</td>
</tr>
<tr>
<td>f</td>
<td>a, b, d, h</td>
</tr>
<tr>
<td>g</td>
<td>a, b, c, d</td>
</tr>
<tr>
<td>h</td>
<td>-b, d, f</td>
</tr>
</tbody>
</table>

**P1 = [ g d h b f a e c ]**
**P2 = [ c e a g b h f d ]**

**O1 = [ e a f b h d c g ]**
**O2 = [ a e c d f b h g ]**

2.2 Mutation Operator

The mutation operator requires a chromosomes string to operate on. The objective of the mutation is to slightly disrupt the current chromosome by inserting a new gene. There are several mutation operators have been proposed for permutation representation, such as reciprocal exchange, inversion and heuristic. In this research we used following mutation operators.
- Reciprocal Exchange Mutation

Random two individuals with the different parents are interchanged and offspring are reproduction.

For instance, with interchanging h and a, turnout offspring.

P1=[g d a b b a c e]

O2=[g d a b f h e c]

- Inversion Mutation

Finding out two cutting points in individuals, inverting the individual’s order between two cutting points and generating offspring.

P1=[g d h b b a c e]

O2=[g d a b f b h e c]

2.3 Performance Comparison of Crossover Operators

There are several crossover operators used in vehicle routing problems such as, OX, OB, PMX, PB, CX. The proposed approach shows the evolution improvement between parents and offsprings across generations relies on the number of parent's good chromosomes that are inherited by the offsprings. The edge information in the parents merges into offspring. Table 2 shows the comparison of the performance of each type of operators for the TSP-type problems.

<table>
<thead>
<tr>
<th>Crossover Class</th>
<th>Type</th>
<th>Fitness for VRP</th>
<th>Fitness for Production &amp; Inventory</th>
<th>Fitness for Mixed Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order Crossover(OX)</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Order Based Crossover(OB)</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Partially Mapped Crossover(PMX)</td>
<td>5</td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Position Based Crossover(PB)</td>
<td>4</td>
<td>1</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Cycle Crossover(CX)</td>
<td>6</td>
<td>3</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Proposed Crossover</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

3. Vehicle Routing Problem with Time Constrained

The vehicle routing problem with limited capacities and travel times from a central depot is formulated as mixed integer program as follows:

- Mathematical Model [8]

\[
\text{Min} \quad \sum_{i=1}^{N} \sum_{j=0}^{K} \sum_{k=1}^{N} C_{ijk} x_{ijk}
\]

Subject to:

\[
\sum_{i=0}^{N} q_{ik} y_{ik} \leq v_k
\]  

\[
\sum_{i=0}^{N} \sum_{j=0}^{N} y_{ik} (t_{ij} + f_i + w_i \leq R_k)
\]

\[
y_{ik} = 0 \text{ or } 1, \quad x_{ijk} = 0 \text{ or } 1
\]

\[
\sum_{k=1}^{K} y_{ik} = 1, \quad i=0, \ldots, N
\]

\[
\sum_{j=0}^{N} x_{ijk} = y_{jk}
\]

\[
\sum_{j=0}^{N} x_{ijk} = y_{ik}
\]

\[
t_j \geq t_i + s_i + t_{ij} - (1 - x_{ijk}) T
\]

\[
f_i \leq t_i \leq l_i
\]

\[
t_i \geq 0, \quad i, j = 1, 2, \ldots N, \quad k = 1, 2, \ldots K
\]

\[
y_{ik} = \begin{cases} 1, & \text{if } i \text{ is serviced by vehicle } k \\ 0, & \text{otherwise} \end{cases}
\]

\[
x_{ijk} = \begin{cases} 1, & \text{if the vehicle } k \text{ travels directly from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}
\]

\[
N = \text{number of customers}
\]

\[
v_k = \text{maximum capacity of vehicle } k
\]

\[
t_{ij} = \text{travel time between customer } i \text{ and } j
\]

\[
t_i = \text{arrival time at customer } i
\]
Table 3. Characteristics of VRP(*: The Study)[1]

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Possible Option</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Size of Available Fleet*</td>
<td>One Vehicle</td>
</tr>
<tr>
<td>2. Type of Available Fleet</td>
<td>Special Vehicle Types(Compartmentalized, etc.)</td>
</tr>
<tr>
<td>3. Housing of Vehicle*</td>
<td>Single Depot(Domicile)</td>
</tr>
<tr>
<td>4. Nature of Demands</td>
<td>Deterministic(known) Demands</td>
</tr>
<tr>
<td>5. Location of Demands</td>
<td>Mixed</td>
</tr>
<tr>
<td>6. Underlying Network*</td>
<td>Euclidean</td>
</tr>
<tr>
<td>7. Vehicle Capacity Restrictions*</td>
<td>Imposed(different Vehicle Capacity)*</td>
</tr>
<tr>
<td>8. Maximum Route Times</td>
<td>Imposed(Different for Different Routes)*</td>
</tr>
<tr>
<td>9. Operations</td>
<td>Drop-offs(deliveries)only*</td>
</tr>
<tr>
<td>10. Costs</td>
<td>Variable or Routing costs*</td>
</tr>
<tr>
<td>11. Objectives</td>
<td>Minimize total routing costs*</td>
</tr>
</tbody>
</table>

Table 3 shows the characteristics for the VRP problems.

For the computational purpose, we developed a computer program by the form of GUI(Graphic User Interface) for the proposed GA approaches. This approach is described below:

1) Set the Initial Values: Gen=0,
   Location of Depot Center : X=0, Y=0,
   Lp-Distance Parameter P=2.0,
2) Construct Initial Population
   Increment the number of iteration, Gen = Gen+1
3) If Gen>Genmax, Stop and Printout, otherwise, goto step 4.
4) Sort the fitness values of each individual of the initial population by ascending order. Construct new offspring by crossover and mutation operators
5) If population size M=number of individuals, N Stop and Printout, otherwise, goto step 2.

The flow of this approach is shown as follow:

![Figure 2. Flow of the proposed approach](image-url)
4. Computational Results

For the purpose of application of proposed model, we developed a GUI-type program for TSP-type problem. A comparison between the solutions obtained by the proposed approach and other GA approaches was made in the following Table 4. For the comparison purpose, the proposed approach described in previous sections has been tested through several TSP-type problems of these TSPs node range from, 0-99 city. Table 4 shows the comparison of the best solutions of three approaches in terms of total travel distance and number of generated. For TSP with large number of cities, the proposed approach seems to be the most feasible approach.

<table>
<thead>
<tr>
<th>Program City</th>
<th>Heuristic 1</th>
<th>Heuristic 2</th>
<th>VRP</th>
<th>GA-TSP</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>150(850)</td>
<td>150(450)</td>
<td>150(319)</td>
<td>132.9(0.431sec)</td>
</tr>
<tr>
<td>30</td>
<td>454(950)</td>
<td>455(950)</td>
<td>451(948)</td>
<td>450.3(1.322sec)</td>
</tr>
<tr>
<td>50</td>
<td>558(1150)</td>
<td>579(4990)</td>
<td>549(1378)</td>
<td>548(2.343sec)</td>
</tr>
<tr>
<td>99</td>
<td>1682(4450)</td>
<td>1719(12450)</td>
<td>1368(1752)</td>
<td>1337.4(5.906sec)</td>
</tr>
</tbody>
</table>

( ) : Generation
Legend : Heuristic 1 : TSP-Solver by Victor M. Kureichik[12]
Heuristic 2 : VRP-Solver by Choi [3]
GA-TSP : GA-TSP model is attain time from optimal solution.

It is known that the proposed approach is potentially efficient and useful in solving TSP-type problem especially for the problem with large number of nodes(city). Figure 3, 4, 5 and 6 show the best solutions of 99-node problem obtained by two existing heuristics and GA-TSP model.
5. Conclusions

In this paper we have described and evaluated a vehicle routing problem with time constrained based on an improved GA operators. The proposed approach is based on an improvement of GA crossover operator by edge-2 crossover. We have developed an improved GUI-based program and applied to TSP-type problems benchmarks range from 10- to 99-city TSPs of varied size of nodes. It was found that the proposed GA approach using improved edge-2 crossover is potentially efficient and useful in solving TSP-type problem.

References


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